NATIONAL TECHNICAL UNIVERSITY OF ATHENS
SCHOOL OF CIVIL ENGINEERING
DEPARTMENT OF STRUCTURAL ENGINEERING
INSTITUTE OF STEEL STRUCTURES

23.06.2016

**LECTURE ANNOUNCEMENT**

Wednesday 6 July 2016, 12:00

Lecture Hall of the Institute of Steel Structures, NTUA

# On the second-order homogenization of wave motion in periodic media and the sound of a chessboard

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## Abstract

This investigation aims to understand the mathematical structure and ramifications of the second-order homogenization of low-frequency wave motion in periodic solids. To this end, multiple-scales asymptotic approach is applied to the scalar wave equation in one and two spatial dimensions. In contrast to previous studies where the second-order homogenization has led to a single fourth-order derivative in the governing equation, present study demonstrates that such asymptotic approach results in a family of field equations uniting spatial, temporal, and mixed (fourth-order) derivatives – that jointly control the incipient wave dispersion. Given the inherent non-uniqueness selecting the affiliated lengthscale parameters, the notion of an optimal asymptotic model is next considered in a one-dimensional setting via its ability to capture the salient features of wave propagation within the first Brillouin zone, including the onset and magnitude of the phononic band gap. From a broader perspective, this result also shows that the formal homogenization approach provides a direct link between the characteristics of the microstructure and the models of gradient elasticity that have been proposed for describing wave propagation in non-local solids.

Considering the wave motion in two dimensions, on the other hand, the asymptotic analysis is first established in a general setting, exposing the constant shear modulus as a sufficient condition under which the second-order approximation of a bi-periodic solid is both isotropic and limited to even-order derivatives. On adopting a chessboardlike periodic structure as a testbed for in-depth analytical treatment, it is next shown that the Mindlin's celebrated long-wavelength approximation of gradient elasticity is incomplete for it does not capture the anisotropic wave dispersion characterized by the "$\sin 4\theta + \cos 4\theta$" term — observed e.g. in chessboard and related periodic structures.

The presentation will conclude with the homogenization of a transmission problem arising in the scattering theory for bounded inhomogeneities with periodic coefficients. The analysis demonstrates that in this case there are two sources of asymptotic perturbation, namely i) the bulk correction — which is described in the main part of this talk, and ii) the boundary correction due to compact support of the periodic structure. For completeness, the study will provide the H1 and L2 estimates of the error committed by the homogenized (scattered field) solution, and highlight the difficulties in characterizing the boundary correction.

## Short Biographical Note of the Speaker

Bojan Guzina obtained his Diploma in Civil Engineering from the University of Belgrade (1989) and M.S. (1992) and Ph.D. (1996) in Civil Engineering from the University of Colorado at Boulder. In 1998 he joined the faculty of the Department of Civil, Environmental, and Geo- Engineering at the University of Minnesota, Twin Cities, where he is now Endowed Shimizu Professor. His research interests are in the areas of inverse scattering, wave motion in periodic and random media, nonlinear waves in tissue-like solids, applied inverse problems in non-destructive evaluation, geophysics, and medicine, mechanics and fracturing of thin films and geodynamics. Among other distinctions, he is Associate Editor of the Journal of Engineering Mechanics, ASCE.

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